摂動的量子色力学による $B \to D_sK$ 崩壊の分岐比の解析

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July 10, 2001 at YITP

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§1. Introduction

1-1 Motivation

\[ B^+ \rightarrow D^0 \pi^+, D^0 \rho^+, D^0 K^+, D^{*0} \pi^+, \]
\[ J/\psi K^+, J/\psi K^*, J/\psi \pi^+, \ldots \]
\[ B^0 \rightarrow D^- \pi^+, D^- \rho^+, D^{*-} \rho^+, J/\psi K^0, \]
\[ K^+ \pi^-, \eta' K^0, K^* \gamma, \ldots \]

many decay modes!

We want to theoretically understand branching ratios (BR), CP asymmetries for all these decay modes.

Some models have been studied to understand them.

Factorization assumption (F.A.) gave predictions to be consistent with the data, while it was unjustified.
1-2 Factorization Assumption (F.A.)

\[
\text{BR}(\bar{B}^0 \rightarrow D^+ \pi^-) \propto \left| \frac{\langle D^+ \pi^- | H_{\text{eff}} | \bar{B}^0 \rangle}{\text{Amp.}} \right|^2 = \text{Amp.}
\]

\[
\text{Amp.} \sim \bar{B}^0 \left\{ \begin{array}{c}
\begin{aligned}
&b \\
&d
\end{aligned}
\end{array} \right\} \begin{array}{c}
\begin{aligned}
&w \\
&u
\end{aligned}
\end{array} \left\{ \begin{array}{c}
\begin{aligned}
&c \\
&d
\end{aligned}
\end{array} \right\} D^+
\]

\[
= \left( \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \right) \left\{ C_2 \langle D^+ \pi^- | (cb) V_{-A}(\bar{d}u) V_{-A} | \bar{B}^0 \rangle + C_1 \langle D^+ \pi^- | (c_i b_j) V_{-A}(\bar{d}_j u_i) V_{-A} | \bar{B}^0 \rangle \right\}
\]

\[
= \left( \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \right) \left\{ a_1 \langle \pi^- | (\bar{d}u) V_{-A} | 0 \rangle \langle D^+ | (cb) V_{-A} | \bar{B}^0 \rangle - i f_{\pi} P_{\pi} \right\}
\]

\[
a_1 \equiv C_2 + \frac{C_1}{N_c} F_{BD} \equiv \langle D^+ | c F_{\pi} (1 - \gamma_5) b | \bar{B}^0 \rangle
\]

\[
= -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* f_{\pi} a_1 F_{BD}
\]

Amp. in F.A.

| \bar{B}^0 \rightarrow D^+ \pi^- | \begin{array}{c}
V_{cb} V_{ud}^* f_{\pi} a_1 F_{BD}
\end{array} |
| B^- \rightarrow D^0 \pi^- | \begin{array}{c}
V_{cb} V_{ud}^* (f_{\pi} a_1 F_{BD} + f_D a_2 F_{BD})
\end{array} |
| \bar{B}^0 \rightarrow D^+ K^- | \begin{array}{c}
V_{cb} V_{us}^* f_K a_1 F_{BD}
\end{array} |

\[
a_1 \text{ and } a_2 \text{ are so determined that BRs agree with the experimental data.}
\]

You can predict the other modes using the same \( a_1 \) and \( a_2 \).

Many BRs agreed with the data.

(Bauer, Stech, and Wirbel, Z. Phys. C34, 103('87))
1-3 $B \to D_sK$ decays

pure annihilation decays

$$H_{\text{eff}}(\Delta B = 1, \Delta C = 1) \sim (\bar{b}c)_{V-A}(\bar{u}d)_{V-A}$$

$$\sim f_B F_{D_sK}$$

$$D_s^- (\bar{c} s), \quad K^+ (\bar{s} u)$$

need a pair creation of $s\bar{s}$

$$\sim \langle D_s^- K^+ | (\bar{u}c)_{V-A} | 0 \rangle \langle 0 | (\bar{b}d)_{V-A} | B^0 \rangle$$

$$\sim f_B F_{D_sK}$$

We can't get $F_{D_sK}$ from experiments.

$\implies$ It's hard to predict $\text{BR}(B \to D_sK)$ in F.A.

(perturbative QCD)

PQCD gives how to calculate $F_{D_sK}$ and non-factorizable contributions

$\implies$ We'd like to predict $\text{BR}(B \to D_sK)$ in PQCD.
$B \rightarrow D_s K$ back-to-back decay

\[ P_K = \frac{m_B}{2}(0.86, 0, 0, -0.86) \quad P_B = (m_B, 0) \quad P_D = \frac{m_B}{2}(1.14, 0, 0, 0.86) \]

\[ \left( \frac{m_K^2}{m_B} \approx 0 \right) \]

\[ p_u \sim O\left(\frac{m_B}{4}\right) \quad d \quad \bar{b} \quad p_c \sim O\left(\frac{m_B}{4}\right) \]

\[ p_{s} \sim O\left(\frac{m_{B}}{4}\right) \quad s \quad p_{s} \sim O\left(\frac{m_{B}}{4}\right) \]

To form $D_s K$ mesons, a pair creation of $s \bar{s}$ quarks is needed.

The energetic $s \bar{s}$ pair can be produced from $q^2 \sim O\left(\frac{m_{B}^2}{4}\right)$ hard gluon.

\[ \Longrightarrow \text{You can treat the contribution of the hard gluon perturbatively.} \]
The decay amplitude in PQCD

\[ B(P_1) \rightarrow \Psi_B \rightarrow D_s(P_2) \rightarrow \Psi_{D_s} \rightarrow K(P_3) \rightarrow \Psi_K \]

\[ k_2 = x_2 P_2 + \vec{k}_{2T}, \]
\[ k_3 = x_3 P_3 + \vec{k}_{3T}, \]

Separating the decay process into hard dynamics \( H \) and wave functions \( \Psi \), the decay amplitudes \( A \) can be calculated.

\[
A \sim \int dx_2 dx_3 b_2 db_2 b_3 db_3 \times \text{Tr}[\Psi_{D_s}(x_2, b_2) H(x_i, b_i) \Psi_K(x_3, b_3)] e^{-S}
\]

- \( k_{iT} \): transverse to \( P_i \), \( b_i \): conjugate space to \( k_{iT} \)

- **hard part \( H \)**: is perturbatively calculated.
  
  (typical energy scale \( \sim \mathcal{O}(M_B) \))

  process dependent

- **wave func. \( \Psi \)**: describes the bound state consisting of the quark and anti-quark. (includes non-perturbative effects)

  It depends on meson, but does not on processes.

The wave functions \( \Psi \) are determined so as for physical quantities to agree with experimental data.

\[ \implies \text{You can predict the physical quantities for the other processes with the same } \Psi. \]
Sudakov factor $e^{-S}$

Identify a meson without distinguishing these diagrams:

\[
\begin{align*}
\begin{array}{c}
\Psi_K & \rightarrow & K \\
& b & \\
\end{array} + \\
\begin{array}{c}
\Psi_K & \rightarrow & K \\
\text{soft gluon} & \rightarrow & \\
\end{array} + \\
\begin{array}{c}
\Psi_K & \rightarrow & K \\
\end{array} + \cdots
\end{align*}
\]

$\Rightarrow$ QCD leads to Sudakov factor that suppresses the probability not to emit the hard gluon.

Because $q\bar{q}$ like to be color singlet, the large $b$ region is suppressed.

This factor makes the perturbative calculation of the hard part more reliable.
QED

\[ \text{not observe} \quad \overset{\text{Sudakov suppression}}{\text{←}} \]

QCD

\[ \text{collinear gluon} \]

\[ \text{hard gluon} \]

\[ \text{\( q \bar{q} \) with the relative separation \( b \):} \]

- \( \frac{\text{small } b}{\text{large } b} \)

- \( \overset{\text{almost color singlet}}{\text{→ cannot emit collinear gluon.}} \)

\[ \overset{\text{always emit collinear gluon}}{\leftrightarrow \text{ suppress } q \bar{q} \text{ state.}} \]

\[ \overset{\text{Sudakov suppression}}{\text{not color singlet}} \]
\section*{3. \(B \rightarrow D_sK\) in PQCD}

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \frac{V_{td}^* V_{ud}}{c_{\beta}} \right) \]

\[ O_1 = (b\bar{d}) V_{td}^* V_{ud} (\bar{u}c) V_{td} A - \frac{1}{2} \]

\[ O_2 = (\bar{b}c) V_{td}^* V_{ud} (\bar{d}u) V_{td} A \]

\[ M_a = f_{B^0} \text{Amp.}(B^0 \rightarrow D_s^- K^+) \]

\[ H(B^0 \rightarrow D_s^- K^+) = \frac{3}{2} \]

\[ f_{B^0} \]
$B$ and $D_s$ meson's wave functions
(the same one used in the previous works)

$$\Phi_{B,\alpha\beta}(x_1, b_1) = \frac{1}{\sqrt{2N_c}} [(P_B \gamma_5)_{\alpha\beta} + M_B \gamma_5 \alpha \beta] \phi_B(x_1, b_1),$$

$$\phi_B(x_1, b_1) = N_B x_1^2 (1 - x_1)^2 \exp \left[ -\frac{M_B^2 x_1^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b_1)^2 \right],$$

$b_1$: conjugate space to the transverse mom. $k_{1T}$

$$\Phi_{D_s,\alpha\beta}(x_2) = \frac{1}{\sqrt{2N_c}} [(\gamma_5 P_{D_s})_{\alpha\beta} + M_{D_s} \gamma_5 \alpha \beta] \phi_{D_s}(x_2),$$

$$\phi_{D_s}(x_2) = \frac{3}{\sqrt{2N_c}} f_{D_s} x_2 (1 - x_2) \{1 + a_{D_s} (1 - 2x_2)\} \sim \frac{f_{D_s}}{f_D} \phi_D(x_2).$$

$K$ meson's wave functions
(given by P. Ball, JHEP 01, 010(1999))

$$\Phi_{K,\alpha\beta}(x_3) = \frac{1}{\sqrt{2N_c}} \left[ \gamma_5 \ P_K \phi_K^A(x_3) + m_0 \gamma_5 \phi_K^P(x_3) + m_0 \gamma_5 (\not\epsilon - \not n - 1) \phi_K^T(x_3) \right]_{\alpha\beta},$$

$$\phi_K^A(x_3) = \frac{f_K}{2\sqrt{2N_c}} 6x_3 (1 - x_3) \left\{ 1 - a_1^K \cdot 3\xi + a_2^K \cdot \frac{3}{2} (-1 + 5\xi^2) \right\},$$

$$\phi_K^P(x_3) = \frac{f_K}{2\sqrt{2N_c}} \left\{ 1 + a_{p1}^K \cdot \frac{1}{2} (-1 + 3\xi^2) + a_{p2}^K \cdot \frac{1}{8} (3 - 30\xi^2 + 35\xi^4) \right\},$$

$$\phi_K^T(x_3) = \frac{f_K}{2\sqrt{2N_c}} (1 - 2x_3) \{1 + a_{T}^K \cdot 3 (-3 + 5\xi^2)\}, \xi = 2x_3 - 1,$$

where $m_0 = \frac{m_k^2}{m_u + m_s}$, $v = \frac{1}{\sqrt{2}} (1, 0, 0, -1) \propto P_K$, $n = \frac{1}{\sqrt{2}} (1, 0, 0, 1)$.

parameters

$0.35$ GeV $\leq \omega_b \leq 0.46$ GeV : parameterize the extent of $B$ meson.

$0 \leq a_{D_s} \leq 1$ : shapes the $D_s$ meson's w.f.

$m_0 = 1.6 \pm 0.2$ GeV : related to the chiral sym. breaking param.

$a_1^K, a_2^K, a_{p1}^K, a_{p2}^K, a_T^K$ : calculated from QCD sum rule.
The current experimental data (PDG) are
\[ \text{BR}(B^0 \to D_s^- K^+) < 2.4 \times 10^{-4}, \]
\[ \text{BR}(B^+ \to D_s^+ \bar{K}^0) < 1.1 \times 10^{-3}. \]

Our results predicted in PQCD

The amplitudes calculated for $\omega_b = 0.4 \text{ GeV}$, $a_{D_s} = 0.3$, and $m_0 = 1.6 \text{ GeV}$ are

<table>
<thead>
<tr>
<th>$B^0 \to D_s^- K^+$</th>
<th>$f_B F_a^{(2)}$</th>
<th>$f_B F_a^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_a$</td>
<td>$M_a'$</td>
</tr>
<tr>
<td>$-8.27 \times 10^{-4} + 1.54 \times 10^{-3}i$</td>
<td>$1.10 \times 10^{-2} - 1.96 \times 10^{-2}i$</td>
<td></td>
</tr>
<tr>
<td>$1.73 \times 10^{-2} - 6.47 \times 10^{-3}i$</td>
<td>$-4.50 \times 10^{-3} - 9.18 \times 10^{-3}i$</td>
<td></td>
</tr>
</tbody>
</table>

\[ a_2 \ll C_2 \implies \text{non-fact.} M_a \text{ is dominant in } B^0 \to D_s^- K^+ \]

\[ \langle \frac{\alpha_s}{\pi} \rangle = 0.11 \implies \text{The perturbative calculations are self-consistent.} \]

The predicted branching ratios are found within,

\[ \text{BR}(B^0 \to D_s^- K^+) = (8.1 \pm 2.0) \times 10^{-5} \]
\[ \times \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{cb}^* V_{ud}|}{0.0402 \cdot 0.9735} \right)^2, \]

\[ \text{BR}(B^+ \to D_s^+ \bar{K}^0) = (3.7 \pm 1.2) \times 10^{-8} \]
\[ \times \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{ub}^* V_{cd}|}{0.0036 \cdot 0.224} \right)^2. \]

This shows that $B^0 \to D_s^- K^+$ is around the corner to be measured by the experiments.
Threshold resummation
resumming the soft and collinear divergences
in order to regulate the hard part $H$.

As for the factorizable diag. at lowest order,
there are higher order corrections.

\[ B \rightarrow D_s^{-}K^{+} \]
give rise to soft & collinear div.
\[ \rightarrow \text{double logarithmic correction} \]

In the factorizable dominant modes,
we can safely neglect the effects of the threshold resummation for non-factorizable contributions.

However,
In $B^0 \rightarrow D_s^{-}K^{+}$ decay mode,
non-factorizable contributions are dominant.

\[ \Rightarrow \text{It remains to investigate the effects of the threshold resummation for non-factorizable contributions.} \]
§4. Summary

We calculate BR for $B \to D_s K$ decays in PQCD, which are caused through the annihilation type diagrams.

We obtain BRs to be consistent with the current data,

- $\text{BR}(B^0 \to D_s^- K^+) \sim 10^{-5} - 10^{-4}$,
- $\text{BR}(B^+ \to D_s^+ \bar{K}^0)$ is smaller than $B^0 \to D_s^- K^+$ by $\mathcal{O}(\lambda^4)$.

For the present, $\text{BR}(B^0 \to D_s^- K^+)$ is close to the upper limit by the experiments.

Although the threshold resummation should be considered, we find that BR for $B^0 \to D_s^- K^+$ are sizeable.