Prospects of the Zee Model

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based on
Y.K. and A. Ghosal, Phys. Rev. D63, 03701 (2001);
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1. Introduction

1.1 Why do the neutrinos have such tiny masses?

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<td>Is there $\nu_R$?</td>
<td>YES</td>
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How about Dirac masses $m_D$?

$m_D \sim m_{e,q}$  
$m_D = 0$

What is a mechanism to reduce the value of $m_D$ to $m_\nu$? to generate such a tiny mass $m_\nu$?

Answer:

1979 GM-R-S, Yanagida  
1980 Zee

Seesaw Mechanism  Radiative Mass

$$M_\nu \sim m_D M_R^{-1} m_D^T$$  

$$M_\nu = \nu_L \begin{array}{c} e^- \end{array} \nu_L$$
1.2 Why we love the Zee model?

A. Zee, PL 93B, 389 (1980)

Zee mass matrix:

\[
M_\nu = \begin{pmatrix}
0 & a & c \\
- a & 0 & b \\
- c & b & 0
\end{pmatrix}
\]  

(1.1)

\[
a = f_{e\mu}(m_\mu^2 - m_e^2) K ,
\]

\[
b = f_{\mu\tau}(m_\tau^2 - m_\mu^2) K ,
\]

\[
c = f_{\tau e}(m_e^2 - m_\tau^2) K .
\]

(1.2)

The model can naturally lead to a large \( \nu \) mixing.


Especially, for the case \( a = c \gg b \), it leads to a bi-maximal mixing

\[
U \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\]  

(1.3)

with

\[
\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq \sqrt{2}b/a .
\]

(1.4)

2. What experimental value is serious for the Zee model?

2.1 A severe constraint on $\sin^2 2\theta_{\text{solar}}$

The Zee model can provide a bi-maximal mixing, but it cannot give the observed sizable deviation from $\sin^2 2\theta_{\text{solar}} = 1$, i.e., $\sin^2 2\theta_{\text{observ}} \sim 0.8$ under the condition $\Delta m_{\text{solar}}^2 / \Delta m_{\text{atm}}^2 \ll 1$.

A parameter independent investigation leads to a severe constraint on the value of $\sin^2 2\theta_{\text{solar}}$

$$\sin^2 2\theta_{\text{solar}} \geq 1 - \frac{1}{16} \left( \frac{\Delta m_{\text{solar}}^2}{\Delta m_{\text{atm}}^2} \right)^2. \quad (2.1)$$

The conclusion cannot be loosened even if we take RGE effects into consideration.

2.2 Outline of the derivation

\[
M_\nu = m_0 \begin{pmatrix}
0 & a & c \\
a & 0 & b \\
c & b & 0
\end{pmatrix}, \quad (2.2)
\]

\[
H_\nu = M_\nu^\dagger M_\nu = H_0 + H_1, \quad (2.3)
\]

\[
H_0 = m_0(|a|^2 + |b|^2 + |c|^2), \quad (2.4)
\]

\[
H_1 = m_0 \begin{pmatrix}
|b|^2 & -c \ast b & -a \ast b \\
-b \ast c & |c|^2 & -a \ast c \\
-b \ast a & -c \ast a & |a|^2
\end{pmatrix}, \quad (2.5)
\]

The eigenvalues of \( H_1 \), \( m_0^2 h_i \), are given by the solutions of the equation

\[
h_i^3 - (|a|^2 + |b|^2 + |c|^2)h_i^2 + 4|a|^2|b|^2|c|^2 = 0, \quad (2.6)
\]

so that the mass spectrum is described only one parameter

\[
|q|^2 = \frac{|a|^2|b|^2|c|^2}{(|a|^2|b|^2|c|^2)^3}, \quad (2.7)
\]
The values \((m_i/m_0)^2\) take \((0, 1, 1)\) and \((1/3, 1/3, 4/3)\) at \(|q|^2 = 0\) and \(|q|^2 = 1/27\), respectively.

The case \(|q|^2 \simeq \frac{1}{27}\) leads to

\[
|U_{\nu 13}|^2 \simeq |U_{\nu 23}|^2 \simeq |U_{\nu 33}|^2 \simeq \frac{1}{3},
\]

so that the case is ruled out.
The case $|q|^2 \simeq 0$ leads to

$$|U_{\nu 23}|^2 \simeq |U_{\nu 33}|^2 \simeq \frac{1}{2}, \quad |U_{\nu 13}|^2 \simeq 0,$$

(2.9)

From the relations

$$(H_\nu)_{ii} = |U_{\nu i 1}|^2 m_1^2 + |U_{\nu i 2}|^2 m_2^2 + |U_{\nu i 3}|^2 m_3^2,$$

(2.10)

We obtain

$$\sin^2 2\theta_{solar} \simeq 1 - \frac{1}{4} \left[1 - 2(|a|^2 - |c|^2)^2\right]^2 |b|^2$$

$$\geq 1 - \frac{1}{16} \left(\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2}\right)^2.$$

(2.11)
2.3 How it is hard to escape from this constraint

♠ Two-loop effects

Chan-Zee, PR D61, 071303 (2000)

⇒ too small

♠ Evolution effects

\( f_{ij}(\mu) \) are dependent on the scale \( \mu \), but the texture \( M_{\nu 11} = M_{\nu 22} = M_{\nu 33} = 0 \) is still kept.

\[
M_e(\Lambda_X) = D_e \quad \Rightarrow \quad M_e(\mu) \text{ is almost kept its diagonal form.}
\]

Therefore, we will be forced optimistically to expect that future experiments will report \( \sin^2 2\theta_{solar} = 1.0 \), or pessimistically to abandon the Zee model.

However, the Zee model is too attractive to be abandoned. We will seek for an extended version with Radiative masses + some additional term.
3 What is a problem in a model with $\sin^2 2\theta_{solar} = 1.0$

3.1 A mystery of the relation $|f_{e\tau}/f_{e\mu}|$

Let us consider that the experiment does not rule out the value $\sin^2 2\theta_{solar} = 1.0$

The bi-maximal mixing is realized only when

$$|M_{\mu 12}| = |M_{\mu 13}| \gg |M_{\mu 23}|,$$  \hspace{1cm} (3.1)

which mean

$$\left| \frac{f_{e\tau}}{f_{e\mu}} \right| \simeq \left( \frac{m_\mu}{m_\tau} \right)^2 \sim 10^{-3},$$ \hspace{1cm} (3.2)

$$\left| \frac{f_{\mu\tau}}{f_{e\tau}} \right| \simeq \frac{1}{\sqrt{2}} \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \sim 10^{-3}. \hspace{1cm} (3.3)$$

The model is highly sensitive to the value of $|f_{e\tau}/f_{e\mu}|$.

Is the relation (3.2) accidental?
3.2 An attempt for acquiring the ratio $|f_{e\tau}/f_{e\mu}|$


We have put a simple ansatz on the transition matrix elements in the infinite momentum frame (not on the mass matrix), and we have obtained the relations

$$f_{ij} = \varepsilon_{ijk} \frac{m^e_k}{m^e_i + m^e_j} f,$$

where $m^e_i = (m_e, m_\mu, m_\tau)$, which leads to

$$
\begin{align*}
a &\propto \left[ m_\tau/(m_\mu + m_e) \right] m^2_\mu \simeq m_\mu m_\tau, \\
b &\propto \left[ m_e/(m_\tau + m_\mu) \right] m^2_\tau \simeq m_e m_\tau, \\
c &\propto -\left[ m_\mu/(m_\tau + m_e) \right] m^2_\tau \simeq -m_\mu m_\tau,
\end{align*}
$$

(3.5)

i.e., the prediction

$$
\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \simeq \sqrt{2}\frac{m_e}{m_\mu} = 6.7 \times 10^{-3}. \tag{3.6}
$$

The predicted value is in excellent agreement with the observed value

$$
\left( \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \right)_{exp} \simeq \frac{2.2 \times 10^{-5} \text{eV}^2}{3.2 \times 10^{-3} \text{eV}^2} = 6.9 \times 10^{-3}. \tag{3.7}
$$
3.3 Open question

The Zee’s Lagrangian is given by
\[ \mathcal{L} = \sum_{i,j} f_{ij} \ell_i^c \tau_2 \ell_j^c L h^- + \sum_i y_i \ell_i L \phi_2 e_i R + \cdots, \]
(3.8)

where the charged lepton mass matrix $M_e$ is diagonal.

If we consider a model
\[ \mathcal{L} = \sum_{i,j} f_{ij}^0 \ell_i^c \tau_2 \ell_j^c L h^- + \sum_{i,j} y_{ij}^0 \ell_i L \phi_2 e_j^0 R + \cdots, \]
(3.9)

with
\[ U_e^\dagger M_e^0 U_e R = M_e \equiv D_e, \]
(3.10)

then, $f_{ij}$ are given by
\[ \begin{pmatrix} f_{23} \\ f_{31} \\ f_{12} \end{pmatrix} = U_{eL}^T \begin{pmatrix} f_{23}^0 \\ f_{31}^0 \\ f_{12}^0 \end{pmatrix}. \]
(3.12)

An open question:

What constraint does the requirement $|f_{12}| \gg |f_{31}| \gg |f_{23}|$ put on the mass matrix model $M_e^0$ and $f_{ij}^0$?
4. How we build a model with \( \sin^2 2\theta_{solar} \sim 0.8 \)?

– Extended versions of the original Zee model –

4.1 Desirable matrix form

\[
M_\nu \simeq \begin{pmatrix}
  c & a & a \\
  a & d & b \\
  a & b & d
\end{pmatrix}, \quad (4.1)
\]

\[
\Delta m_{solar}^2 \propto b - (c + d) \simeq 0, \quad (4.2)
\]

\[
\tan 2\theta_{solar} \simeq 2\sqrt{2} \frac{a}{c - d + b}, \quad (4.3)
\]

\[
U_{e3} = 0. \quad (4.4)
\]

Fukuyama-Nishiura, (1997);

4.2 How to escape from the constraint 
\[ \sin^2 2\theta_{solar} = 1.0 \]

Two-loop effects in a model with a new doubly charged scalar \( k^{++} \)

- Zee (1986), Babu (1988), · · ·;

Yukawa coupling of the charged leptons to both scalars \( \phi_1 \) and \( \phi_2 \)


\( R \)-parity violating SUSY model

We identify Zee scalar \( h^+ \) as Slepton \( \tilde{e}_R \):

\[ \Rightarrow \text{ additional contributions from } d \text{-quark loops} \]

- Dress, et al., (1998); and many authors.
5. Concluding remarks

1. The Zee model can provide us rich phenomenology.

\[ e_i^- \rightarrow e_j^- \gamma, \quad Z \rightarrow e_i^\pm e_j^{\mp}, \quad h^0 \rightarrow \gamma \gamma, \ldots \]

2. There is a serious problem in the Zee model

\[ \sin^2 2\theta_{solar} \geq 1 - \frac{1}{16} \left( \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \right)^2. \]

We will seek for an extended version.

3. Attempts to embed the Zee model into a GUT scenario

In future, such investigations will become more important in order to be free from the sever constraint on the original Zee model.